4.1.A.

The expression for the transverse magnetization is given by:

$$M_{xy}(TR,TE) = M_0(1-e^{-TR/T_1})e^{-TE/T_2}$$

The maximum in the parameter of interest, $\Delta M_{xy} = |M_{xy,tissue1} - M_{xy,tissue2}|$, can be found by taking the derivative with respect to TE, giving TE = 38.3 ms.

B.

Following similar arguments as under A it can be shown that a local maximum in the signal difference intensity can be found at TR = 786 ms. However, this does not correspond to the maximum attainable difference which is obtained when TR $\rightarrow \infty$.

C.

Tissue contrast disappears when $M_{xy,tissue1} = M_{xy,tissue2}$. For the given parameters, this occurs when TE = 10.78 ms.

4.2.A.

256 complex acquisition points over 1.28 ms equals a dwell-time of (1280/256) = 5 μs, which in turn gives a bandwidth of 200 kHz. The readout gradient of 2.4466 G/cm corresponds to 10.415 kHz/cm, such that the field-of-view in the frequency encoding direction becomes (200/10.415) = 19.2 cm.

B. Since all gradient ramps take 500 μs, the phase encoding gradient is dictated by a trapezoidal shape with a 500 μs up-going ramp, a 200 μs plateau period and a 500 μs down-going ramp. This phase-encoding gradient would be equivalent to a 700 μs constant-amplitude gradient (i.e. infinitely fast gradient switching) of the same amplitude. A modification of Eq. [4.19] can be constructed for the phase-encoding direction according to:

$$\text{FOV} = \frac{1}{\Delta k} = \frac{1}{\Delta G t} \qquad [s4.1]$$

where FOV and k are in units of cm and cm^{-1}, respectively, and ΔG and t are in units of Hz/cm and s, respectively. Eq. [s4.1] thus yields phase-encoding increments of ΔG = 74.4 Hz/cm = 0.0175 G/cm.

C.

Slice selection gradient amplitude = RF bandwidth/slice thickness = 3750/0.4 = 9375 Hz/cm = 2.2 G/cm.

D.

In order to calculate the minimum echo-time, the slice selection refocusing and the readout preparation gradient durations need be known. Both gradient durations can be minimized by executing them at maximum (negative) amplitude, which will lead to plateau durations of 380 μs and 371 μs for the slice selection refocusing and readout preparation gradients, respectively. Since the phase-encoding plateau duration is only 200 μs, it is the slice selection gradient that dictates the minimum echo-time. Under the requirement that, besides the desired gradients, no additional (orthogonal) gradients should be present during the excitation and acquisition periods, the minimum echo-time is given by 3.27 ms.

4.3.A.

1. experimental duration = 32 × 3 s = 96 s. All 32 slices can be accommodated within the repetition time TR.
2. experimental duration = 32 × 32 × 0.01 = 10.24 s. The repetition time is based on one slice, such that 32 slices take 320 ms.
3. experimental duration = 4 × 3 s = 12 s. All 32 slices, as well as 8 phase-encoding increments can be accommodated within the repetition time TR.

The gradient-echo sequence is the fastest method for the selected parameters.

B.

1. experimental duration = 128 × 3 s = 384 s. All 128 slices can be accommodated within the repetition time TR, since 128 × TE < TR.

2. experimental duration = 128 × 128 × 0.01 = 164 s. The repetition time is based on one slice, such that 128 slices now take 1280 ms.

3. experimental duration = 32 × 3 s = 96 s. All 128 slices, as well as 8 phase-encoding increments can be accommodated within the repetition time TR.

The EPI sequence is the fastest method for the selected parameters.

4.4.

A.

- Increasing the field-of-view to 21.6 × 21.6 cm improves the SNR 1.266 times.
- Increasing the number of averages to 2 improves the SNR √2 = 1.414 times.
- Decreasing the nutation angle to 15º improves the SNR 1.147 times.
- Increasing the repetition time to 100 ms improves the SNR 2.447 times.

Therefore decreasing the nutation angle and increasing the repetition times give the smallest and largest absolute improvements in SNR.

B.

The standard experiment produces a SNR of 40 over 96 × 25 ms = 2.4 s which equals 16.67 SNR/s.

- Increasing the field-of-view to 21.6 × 21.6 cm provides 21.1 SNR/s.
- Increasing the number of averages to 2 provides 11.8 SNR/s.
- Decreasing the nutation angle to 15º provides 19.1 SNR/s.
- Increasing the repetition time to 100 ms provides 10.2 SNR/s.

Increasing the field-of-view thus provides the highest SNR per unit of time, of course at the cost of a lower spatial resolution.

C.

The Ernst angle of 10.43º gives 1.225 times more SNR than an experiment executed with $\alpha = 20^\circ$.

4.5

A.

$$M_z = M_0 \frac{1-(e^{-TR/T_1} + e^{-(n-1)TR/T_1}\cos\alpha_2)(1-\cos\alpha_2) - e^{-nTR/T_1}\cos^2\alpha_2}{1-e^{-nTR/T_1}\cos\alpha_1\cos^2\alpha_2}$$

B.

$$M_z = M_0 \frac{1-(e^{(1-n/2)TR/T_1} + e^{-nTR/2T_1}\cos\alpha_2)(1-\cos\alpha_2) - e^{-nTR/T_1}\cos^2\alpha_2}{1-e^{-nTR/T_1}\cos\alpha_1\cos^2\alpha_2}$$

C.

Sequential, $M_z = 0.8626M_0$

Interleaved, $M_z = 0.8670M_0$

D.

Sequential, $M_z = 0.7408M_0$

Interleaved, $M_z = 0.8000M_0$

4.6.

A.

The nutation angle θ can be obtained with the aid of Eq. [4.31] and is equal to 68º.

B.

From the calibration it follows that $\theta_{\text{total slice}} = 0.818\theta_{\text{on-resonance}}$. Therefore, since $\theta_{\text{total slice}} = 68^\circ$ the on-resonance nutation angle becomes $\theta_{\text{on-resonance}} = 83^\circ$. Note that under practical conditions, the on-resonance nutation angle is in general not linearly related to the effective nutation angle over the entire slice. In those cases, the on-resonance nutation angle can be calculated through numerical simulation.

4.7.

A.

Only the phase difference between the delays is important for the calculation of a frequency offset. Therefore:

$\tau_2 - \tau_1 = 1.0$ ms gives a phase difference of $+76^\circ - (-86^\circ) = +162^\circ$

$\tau_3 - \tau_1 = 2.0$ ms gives a phase difference of $-128^\circ - (-86^\circ) = -42^\circ$

$\tau_4 - \tau_1 = 4.0$ ms gives a phase difference of $-171^\circ - (-86^\circ) = -85^\circ$

Using only the first two data points gives:

$$\Delta\nu = \frac{\Delta\phi}{360^\circ \Delta\tau} = \frac{162^0}{360^\circ 0.001 \text{ s}} = +450 \text{ Hz}$$

Using all points requires linear regression of the ($\Delta\tau$, $\Delta\phi$) curve. However, linear regression can only be performed when all phases are properly phase-unwrapped. Based on the estimation provided by the first two data points, the phase difference between the first and third points can be estimated as 324º. Given the fact that the measured phase difference is –42º it is clear that a –360º phase wrap has occurred. The real phase difference between the first and third points should thus be $-42^\circ + 360^\circ = +318^\circ$. Similarly, the phase difference between the first and fourth points can be recalculated as $-85^\circ + 2 \times 360^\circ = +635^\circ$.

Linear regression of $\Delta\tau = \{1.0, 2.0, 4.0\}$ ms and $\Delta\phi = \{+162^\circ, +318^\circ, +635^\circ\}$ then gives $\Delta\nu = 438.3$ Hz.

The frequency offset calculated with all delays has a higher sensitivity as a wider range of evolution delays and more points are used.

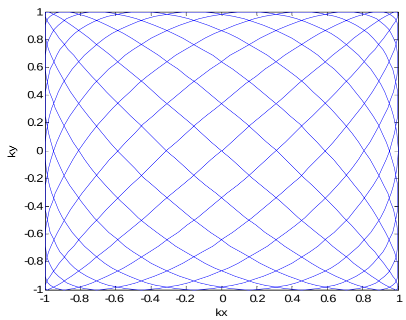
B.

The spatial displacement in the phase-encoding direction is zero.

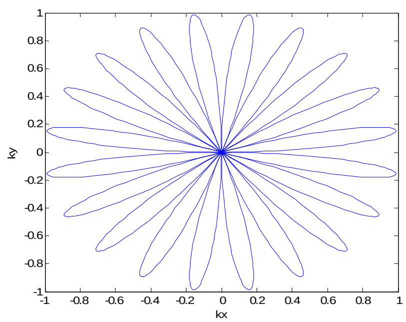
In the readout direction, the bandwidth per pixel is 50 kHz/64 = 781.3 Hz. Therefore, a frequency offset of +400 Hz leads to a shift of 0.512 pixels.

4.8.

A.



B.



C. - Both trajectories do not allow sampling on a rectangular grid and hence one needs to perform a regridding procedure before FT can be applied.

- Both trajectories have non-uniform k-space sampling, The Lissajou trajectory has increased density at high k-space coordinates and the Rosette trajectory at low k-space coordinates.

- Especially for the Lissajou trajectory, the non-uniform density leads to a unfavorable point spread function.

4.9.A.

$$\theta = \arccos\left(\frac{M_{xy,2}}{M_{xy,1}}\right)$$

B.

The error in the nutation angle can be obtained through standard error propagation theory according to:

$$\sigma_\theta^2 = \sigma_{M_{xy,1}}^2 \left(\frac{\partial\theta}{\partial M_{xy,1}}\right)^2 + \sigma_{M_{xy,2}}^2 \left(\frac{\partial\theta}{\partial M_{xy,2}}\right)^2$$

which evaluate to

$$\sigma_\theta^2 = \sigma^2\left(\frac{M_{xy,1}^2 + M_{xy,2}^2}{M_{xy,1}^4 - M_{xy,1}^2 M_{xy,2}^2}\right) = \sigma^2\left(\frac{1+\cos^2\theta}{\sin^4\theta}\right)$$

when equal noise levels in the two experiments are assumed.

The minimum in σ^2_θ can be found by taken the derivative with respect to θ and is found when θ = 90º.

C.

The optimal nutation angle for the standard method also equals 90º. However, the absolute error is 2 times smaller than for the faster alternative (or $2^{1/2}$ smaller for equal scan times).

4.10A. From Eq. (4.15) it follows that a suppression factor f is obtain when:

$$\frac{1}{f} = \frac{2}{\gamma L G t}$$, which results in γG = 20 kHz/cm.

B. f = 6.25.

4.11.A.

$$TR=\frac{\ln\left(\frac{T_{1A}}{T_{1B}}\right)}{\left(\frac{1}{T_{1B}}-\frac{1}{T_{1A}}\right)}$$

B.

$$TE=\frac{\ln\left(\frac{T_{2A}}{T_{2B}}\right)}{\left(\frac{1}{T_{2B}}-\frac{1}{T_{2A}}\right)}$$

4.12.

A.

Solving the first equation for T_1 gives the general expression

$$T_1=\frac{TR1}{\ln\left[\frac{S_0}{S_0-S_1}\right]}$$

Using the fact that $TR1/T_1$ is identical for both equations gives the expressions for S_0, which are dependent on TR2.

For TR2 = 2TR1, S_0 is given by

$$S_0=\frac{S_1^2}{2S_1-S_2}$$

For TR2 = 3TR1, S_0 is given by

$$S_0 = \frac{3S_1^2 + S_1\sqrt{4S_1S_2 - 3S_1^2}}{6S_1 - 2S_2}$$

B.

Similar to exercise 4.9B the error in T_1 can be determined by standard error propagation and is given by:

$$\sigma_{T_1}^2 = \sigma^2 \left(\frac{TR^2 (S_1^2 + S_2^2)}{S_1^2 (S_2 - S_1)^2 \ln\left(\frac{S_1}{S_2 - S_1}\right)^4} \right)$$

While the optimal TR can be obtained by setting the derivative of σ^2_{T1} with respect to TR to zero, a much simpler method is to plot this equation. The smallest error for a T_1 of 1000 ms is obtained when TR = 1572 ms.

4.13.

A.

Readout acquisition time = 64/100 kHz = 640 μs.

The time between two phase-encoding increments thus becomes 100 + 640 + 100 μs = 840 μs, making the effective spectral width in the phase encoding direction 1190 Hz.

B.

In general the spatial displacement (in pixels) equals the frequency offset divided by the spectral bandwidth per pixel.

In the readout direction, the spectral bandwidth per pixel is (SW/N_f), whereas in the phase-encoding direction, the effective spectral bandwidth is ($SW/(N_pN_f)$). Dividing the frequency offset $\Delta\nu$ by the spectral bandwidths gives Eq. [4.21].

C.

Pixel shift in frequency encoding direction = 0.096

Pixel shift in phase encoding direction = 8.067

4.14.A. $\phi = \frac{2\pi}{3}k$, where k represents the k-space coordinate. k = 0 in the middle of k-space.

B. $\phi = 2\pi x$ with $-FOV/2 \le x \le +FOV/2$

C.

$SNR_{downsampling} = \sqrt{4}\ SNR_{original} = 2\ SNR_{original}$

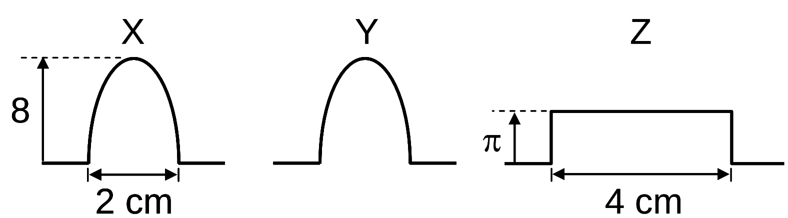
For the same amount of acquisition time, the SNR for the newly acquired image is:

$SNR_{new} = 4\ SNR_{original}$.

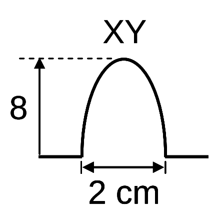
In order to obtain the highest SNR it is therefore always recommended to acquire the image at the desired resolution.

4.15.

A.



B.



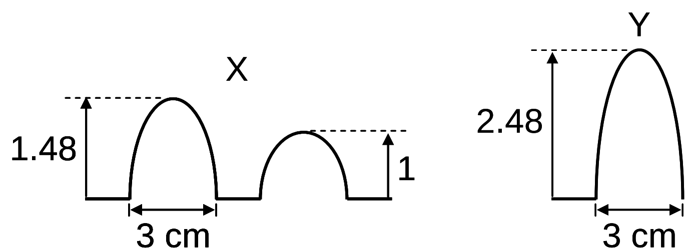
C.

Spectral width = 1 MHz over the field-of-view or 3,906 Hz per pixel. Given that the chemical shift difference between water and methanol at 4 T is (4.7 – 3.3) × 170.2 = 238 Hz, the chemical shift displacement between the two tubes is less than 0.1 pixel and can thus be ignored. Since a spin-echo sequence refocuses all phase evolutions, the spatial profiles are dictated by proton density differences.

1 mL water = 1 gram water = 0.111 mol water protons.

1 mL methanol = 0.8 gram methanol = 0.075 mol methanol methyl protons.

Therefore the intensity ratio between the two tubes is 1.48.



D. Phase evolution is not refocused during a gradient-echo sequence, such that a phase difference of $2\pi \times 238$ Hz $\times$ 6.3 ms = 3π is generated between the water and methanol signals. While an absolute-valued profile would be identical to that sketched under C, a phase-sensitive profile would display a negative signal intensity of the methanol tube relative to the water tube.

E. Spectral width = 20 kHz over the field-of-view or 78.1 Hz per pixel. The chemical shift difference between water and methanol at 11.7 T is (4.7 – 3.3) × 500 = 700 Hz, such that the chemical shift displacement is equal to ~ 9 pixels or 0.35 cm. The direction of the shift depends on the sign of the frequency-encoding gradient.