

### 5.1.

#### A.

$\gamma B_1 = ((\pi/2)/(2\pi T)) = 1/(4T) = 250 \text{ Hz}$ .  $\gamma = 4257 \text{ Hz/G} = 42.57 \text{ Hz}/\mu\text{T}$ , therefore  $B_1 = (250/42.57) = 5.87 \mu\text{T}$ .

#### B.

$B_0 = 3 \text{ T} = 127.66 \text{ MHz}$  or  $127.66 \times 10^6$  revolutions per second, therefore in 1 ms there will be 127,660 revolutions.

5.2.  $\theta = \gamma B_1 T$ , therefore  $T = \theta/\gamma B_1 = \pi/(2\pi \times 42.57 \text{ MHz/T} \times 37.6 \mu\text{T}) = 312.4 \mu\text{s}$ .

### 5.3.

#### A.

$R_{90} = 6.08$  and  $R_{180} = 12.26$ , making the bandwidths 6.08 and 6.13 kHz for 1 and 2 ms pulses, respectively.  $G_{90} = 6.08 \text{ kHz}/0.2 \text{ cm} = 30.4 \text{ kHz/cm}$  and  $G_{180} = 6.13 \text{ kHz}/0.2 \text{ cm} = 30.65 \text{ kHz/cm}$ .

#### B.

Total positive gradient duration =  $0.5 + 0.15/2 = 0.575 \text{ ms}$ , and amplitude 30.4 kHz/cm. Total negative gradient duration =  $0.15/2 + 0.5 + 0.15/2 = 0.65 \text{ ms}$ . Therefore, negative gradient amplitude =  $-(0.575 \times 30.4/0.65) = -26.9 \text{ kHz/cm}$ .

#### C.

Across the excitation profile where  $M_{xy} \geq 0.5M_0$  (i.e.  $-9.00 \leq \nu \leq +9.00 \text{ kHz}$ ) there exists a circa  $500^\circ$  phase roll due to phase evolution during the 1 ms pulse. This corresponds to a free precession period T of  $(250^\circ/(360^\circ \times 9.00 \text{ kHz})) = 0.077 \text{ ms}$  (as compared to  $\sim 0.5 \text{ ms}$  for a linear SLR pulse).

$R_{90} = 18.74$ , making the slice gradient strength =  $18.74 \text{ kHz}/0.2 \text{ cm} = 93.7 \text{ kHz/cm}$ . Total positive gradient duration =  $0.077 + 0.15/2 = 0.152 \text{ ms}$ . Therefore, negative gradient amplitude =  $-(0.152 \times 93.7/0.65) = -21.9 \text{ kHz/cm}$

#### 5.4.A.

$M_z/M_0 = \cos^2\theta\cos2\theta - \sin^2\theta$  where  $\theta = \gamma B_1 T =$  excitation pulse angle.

#### B.

Solving  $M_z/M_0 = \cos^2\theta\cos2\theta - \sin^2\theta = -0.98$  for  $\theta$  gives the range  $71.6^\circ < \theta < 108.4^\circ$ .

#### C.

The compensation originates from that fact that the middle pulse  $2\theta_{+y}$  rotates any remaining  $M_z$  to the opposite side of the transverse plane, giving rise to the  $\cos2\theta$  term as derived under A. Without the middle pulse, the expression would reduce to that for a simple square inversion pulse (i.e.  $\cos2\theta$ ).

#### D.

For a hard pulse the inversion interval is encountered for  $84.3^\circ < \theta < 95.7^\circ$ . MLEV is therefore  $(108.4^\circ - 71.6^\circ)/(95.7^\circ - 84.3^\circ) = 3.22$  times less sensitive to error in the nutation angle.

E. The composite WALTZ pulse provides excellent compensation towards off-resonance effects. However, it does not provide any compensation with respect to RF inhomogeneity.

#### 5.5.

A.  $SAR \sim B_{1max}^2 \cdot f_{B,average} \cdot T = B_{1rms}^2 T$

Doubling the length T, halves the  $B_{1rms}$ , such that the power deposition reduces by 50%.

#### B.

From Table 5.1 it follows that:

$B_{1rms}(\text{sinc}, 90) = 1.52 \text{ kHz}$

$B_{1rms}(\text{sinc}, 180) = 3.04 \text{ kHz}$

$$B_{1\text{rms}}(\text{SLR}, 90) = 1.38 \text{ kHz}$$

$$B_{1\text{rms}}(\text{SLR}, 180) = 3.85 \text{ kHz}$$

$$\text{Relative SAR with sinc pulses} : (1.52)^2 + 2 \times (3.04)^2 = 20.79$$

$$\text{Relative SAR with SLR pulses} : (1.38)^2 + 2 \times (3.85)^2 = 31.55$$

The SLR pulses thus increase the SAR by more than 50%.

### C.

A 5 ms square 90° pulse requires  $B_1 = 50$  Hz. Since the maximum RF amplitude is limited to 1500 Hz only those pulses in Table 5.1 with  $B_{1\text{max}} < 30$  can be selected. In order to minimize the chemical shift displacement, the bandwidth =  $R/T$  must be maximized. Since the shortest pulse length  $T$  can be achieved with a pulse that requires the lowest  $B_{1\text{max}}$ , such that maximizing  $R/T$  is synonymous with maximizing  $R/B_{1\text{max}}$ . When the square pulse is excluded (on grounds of the poor frequency profile), the best pulses for excitation and refocusing are the Gaussian (10% truncation) and regular sinc pulses, respectively. It therefore follows that the optimized RF pulses do not necessarily improve the bandwidth per unit of  $B_1$ . Instead they improve the shape of the frequency profile.

### D.

The 90° Gaussian pulse can be executed with  $T = 0.295$  ms and  $B_{1\text{max}} = 1.5$  kHz, while the 180° sinc pulse can be executed with  $T = 1.9$  ms and  $B_{1\text{max}} = 1.5$  kHz.

$$\text{The relative SAR of the default pulses is: } (1.52)^2 + (3.04)^2 = 11.552$$

The relative SAR of the new, maximum bandwidth pulses is:

$$(2.0/0.295)(1.00)^2 + (2.0/1.9)(3.04)^2 = 16.508$$

### E.

Reducing the echo-time does not change the SAR. Increasing the TR by a factor of 2 decreases the SAR by a factor of 2.

## 5.6.

### A.

The reduction in length and the increase in nutation angle both increase the RF amplitude requirements by a factor of 2. Therefore, depending on whether the amplifier setting is implemented as an attenuator or amplifier, the power setting should be  $23 - 12 \text{ dB} = 11 \text{ dB}$  or  $23 + 12 \text{ dB} = 35 \text{ dB}$ .

### B.

Using Table 5.1, a 1 ms 90 sinc pulse require  $B_{1\text{max}} = 1425 \text{ Hz}$ . The power setting for this pulse must therefore be set to  $10 \pm 20\log_{10}(1310/1425) = 10 \pm 0.73 \text{ dB}$ , depending on the mode of the amplifier.

**5.7.** A 10 ms AFP pulse with  $R = 60$  has a bandwidth of 6.0 kHz.

**A.** At an offset of +30 kHz the magnetization tilts away from the +z axis during the pulse, after which it will return to the +z axis at the end of the pulse. In the presence of T2 relaxation, part of the magnetization may disappear during the pulse so that not all magnetization return to the +z axis.

**B.** At a frequency offset of +3 kHz the frequency modulation of the pulse can be seen as running from +6 kHz to 0 kHz. This means that at the end of the AFP the magnetization is excited, rather than inverted.

**C.** At a frequency offset of +0.3 kHz the magnetization inverts to the -z axis. In the presence of T<sub>1</sub> relaxation the inversion will be incomplete.

**D.** At a frequency offset of -3 kHz the frequency modulation of the pulse can be seen as running from 0 kHz to -6 kHz. At the point the magnetization also is excited.

**E.** The features under B and D can be used for frequency selective excitation, for example in water suppression. The features in A and C, i.e. frequency-selective inversion has numerous application ranging from spatial localization to spectral editing. None of the features A-D are present during a AFP pulse based on tanh/tan modulation, since the frequency profile of this pulse is dependent on the B<sub>1</sub> amplitude.

**5.8.** Consider a 16.02 ms optimized SLR 90° pulse ( $R = 6.0$ ), segmented into 81 RF pulses of 20  $\mu\text{s}$  each interleaved with 180  $\mu\text{s}$  delays.

**A.**

The width of the center band of any DANTE sequence is simply given by  $R/T$ , where  $T$  is the duration of the entire pulse. Therefore, the bandwidth of the center band equals  $6/16.02 = 374.5$  Hz.

**B.**

Adjacent excitation bands will appear at  $n/(20 \mu\text{s} + 180 \mu\text{s}) = n \cdot 5.0$  kHz ( $n = 1, 2, 3 \dots$ ).

**C.**

Relative SAR conventional pulse =  $16.02 (B_1)^2$

Relative SAR DANTE pulse =  $1.62 (B_1 \cdot 16.02/1.62)^2 = 158.4 (B_1)^2$

**D.** Equal separation between the excitation bands means that the sum of one pulse segment and one delay must remain constant at 200  $\mu\text{s}$ . Halving the excitation band means that the pulse length must be doubled. The new pulse therefore consists of 161 pulse segments of 20  $\mu\text{s}$  interleaved with 160 delays of 180  $\mu\text{s}$  each. (This brings the pulse to within 0.1% of the requested parameters).

**5.9.** A PRESS sequence is executed with 3 ms optimized SLR90 ( $R = 12$ ) and SLR180 ( $R = 12$ ) excitation and refocusing pulses, respectively. Signal is acquired from a  $2 \times 2 \times 2$  cm = 8 mL volume in the human occipital cortex with  $TR = 1500$  ms and  $TE = 20$  ms.

**A.**

A 3 ms SLR90 ( $R = 12$ ) pulse has a 4.0 kHz bandwidth. The required gradient strength to select a 2 cm volume is then equal to 2.0 kHz/cm. The amplitude of the refocusing gradient can then be calculated as:

$$G_{\text{refocusing}} = -2.0 \times (3000/2 + 500/2) / (500/2 + 1000 + 500/2) = -2.33 \text{ kHz/cm}$$

The first factor of 2 in 3000/2 comes from the fact that only half of the pulse length needs to be refocused. The other factors of 2 come from the fact that gradient ramps equal half the area of full amplitude gradients.

**B.**

With a constant-slope gradient, the ramp times will become dependent on the gradient amplitude. If GR is the gradient ramping rate (in kHz/cm/ms), then the gradient ramp duration,  $t$ , to a gradient amplitude  $G$  is equal to  $G/GR$ . For slice selection and refocusing gradient amplitudes  $G_{SS}$  and  $G_{SSR}$ , pulse length  $T$  and gradient refocusing plateau duration  $T_{\text{plateau}}$ , the gradient areas are equal when:

$$\left(\frac{T}{2} + \frac{G_{SS}}{GR}\right)G_{SS} + \left(\frac{G_{SSR}}{GR} + T_{\text{plateau}}\right)G_{SSR} = 0$$

which gives  $G_{SSR} = -3.47$  kHz/cm.

**C.**

With an effective dephasing pulse length of  $T/10$ ,  $G_{SSR}$  now becomes  $-0.733$  kHz/cm for both modes of gradient operation.