6.1.A.

RF excitation bandwidth = 6080/2 = 3040 Hz

RF refocusing bandwidth = 5750/4 = 1438 Hz

A magnetic field of 4 Tesla corresponds to 170.2 Hz/ppm, such that the choline and creatine resonances appear at 205.9 Hz and 173.8 Hz relative to the NAA resonance, respectively. Relative to NAA, the volume for choline has a spatial displacement of 0.20 cm (= 6.8 %) and 0.43 cm (= 14.3 %) in the spatial directions corresponding to the excitation and refocusing pulse respectively. Creatine has displacements of 0.17 cm (= 5.7 %) and 0.36 cm (= 12.1 %), respectively.

As a result, the 3D volumes for choline and creatine overlap for 68.4% and 72.9% with the NAA volume, respectively.

B.

From Table 5.1 it follows that a 2 ms linear SLR excitation pulse requires B_{1max} = B_{1max}(square) × 6.15 = 769 Hz. A 4 ms optimized sinc refocusing pulse requires B_{1max} = 1363 Hz.

With a maximum achievable B_{1max} of 2500 Hz, the excitation and refocusing pulse length can therefore be reduced to 0.615 ms and 2.18 ms, respectively, making the corresponding RF bandwidths equal to 9886 Hz and 2638 Hz, respectively. The required gradient strengths to select a 3 cm volume then become 3295 Hz/cm and 879 Hz/cm. Both gradient strengths are below the maximum achievable strength of 20 mT/m = 8512 Hz/cm, such that the RF amplitude rather than the gradient strength is the limiting factor.

With the new RF bandwidths the spatial displacements for choline become 0.06 cm (= 2.1 %) and 0.23 cm (= 7.8 %), respectively and for creatine become 0.05 cm (= 1.8 %) and 0.20 cm (= 6.6 %), such that the total volume overlap become 83.2 % and 85.7%, respectively.

C.

Lactate has two distinct chemical groups resonating at 1.313 ppm and 4.097 ppm. The frequency difference between these two resonances at 4.0 T equals 473.8 Hz. For a RF refocusing bandwidth of 1438 Hz this corresponds to a chemical shift displacement of 32.95%. For a 2D volume selected by the 180º pulses, this means that in only 44.96% of the volume both resonances are refocused by both pulses. In 44.18% of the volume only one of the two refocusing pulses refocuses both resonances, while in 10.86% of the volume only the methyl resonance is refocused by both pulses.

Given the fact that TE = 274 ms, this means that the lactate methyl signal is positive in-phase in 44.96% of the volume, negative in-phase in 44.18% of the volume and positive in-phase in the remaining 10.86%. Since the lactate methyl signal would be 100% positive in-phase in the absence of chemical shift displacement, the observed lactate methyl signal intensity is +44.96 – 44.18 + 10.86 = 11.64% of the maximum.

A similar result could have been obtained through the use of Eqs. [6.11]-[6.17].

D.

The calculation for TE = 548 ms is identical to that given under Exercise C, with the exception that the lactate methyl signal is positive in-phase for all sub-volumes, such that 100% lactate is observed despite the presence of a chemical shift displacement. It should of course be realized that the longer echo-time may have greatly diminished the lactate methyl intensity due to T_2 relaxation losses.

6.2.

A.

The expression for the magnetization following the second crusher gradient is given by:

$$M_z = M_0 - M_0(1 - \cos\alpha[1 - e^{-\Delta/T_1}])e^{-\Delta/T_1}$$

where α and Δ are the nutation angle of the second RF pulse and the crusher gradient duration, respectively. Ideally $M_z = 0$, but it can readily be shown that no solutions exist for $M_z = 0$, such that perfect suppression is not possible! The optimal nutation angle can be found with $(dM_z/d\alpha) = 0$, which gives

$$\sin\alpha = 0$$

and thus $\alpha = 0^o + n180^o$ (n = integer) where $0^o + n360^o$ corresponds to the maxima and $180^o + n360^o$ corresponds to the minima (i.e. the best suppression). Note that the optimal nutation angle is independent of T_1.

B. For the ideal sequence $90^o – \Delta – 180^o – \Delta$, $(M_z/M_0) = 2.5 \times 10^{-4}$.
When the B_1 field is off by –10%, the sequence becomes $81^o – \Delta – 162^o – \Delta$ leading to $(M_z/M_0) = –0.143$. When the B_1 field is off by +10%, the sequence becomes $99^o – \Delta – 198^o – \Delta$ leading to $(M_z/M_0) = +0.145$. The suppression is thus highly dependent on the B_1 amplitude and the sequence will not provide adequate water suppression when used with coils producing an inhomogeneous B_1 field.

C.

Residual water proton signal in gray matter = 0.6 × 50 M × 2 protons/molecule × (M_z/M_0) = 15.12 mM protons.

Residual water proton signal in white matter = 0.4 × 48 M × 2 protons/molecule × (M_z/M_0) = 13.67 mM protons.

Total water proton signal = 15.12 + 13.67 = 28.79 mM.

Total creatine signal = 10 mM × 3 protons/molecule = 30 mM.

Water intensity relative to creatine methyl intensity = 0.96.

D.

Miss calibration of the B_1 field by –10%:

Residual water proton signal in gray matter = –8585 mM

Residual water proton signal in gray matter = –5451 mM

Water intensity relative to creatine methyl intensity = –468.

Miss calibration of the B_1 field by +10%:

Residual water proton signal in gray matter = 8707 mM

Residual water proton signal in gray matter = 5548 mM

Water intensity relative to creatine methyl intensity = 475.

6.3.

A.

From Chapter 2 (section 2.2.38) it follows that $\Delta\omega$ = 0.15 ppm which equals 9.6 Hz at 1.5 T and J = 15.5 Hz. The maximum echo-time is then given by TE_{CPMG} = 10.96 ms.

B.

At 7.0 T, $\Delta\omega$ becomes 45 Hz, such that the maximum echo-time TE_{CPMG} = 4.20 ms.

C.

From Table 2.1 it follows that $\Delta\omega$ = 2.784 ppm which equals 835.2 Hz at 7.0 T and J = 6.93 Hz. The maximum echo-time is then given by TE_{CPMG} = 0.24 ms. This example shows that the CPMG condition is easily fulfilled for strongly coupled spin-systems, like citrate, but that the echo-time becomes unattainably short for weakly-coupled spin-systems, such as lactate, at higher magnetic fields.

6.4.A.

$$M_z(n) = M_0 - \sum_{k=1}^{n} M_0 (\cos\alpha)^{k-1}(1-\cos\alpha)e^{-k\Delta t/T_1}$$

B.

The expression for the TR period of STEAM is identical to that derived for PRESS under A. Unlike the TR period, T_1 relaxation during the TM period actually helps the water suppression according to:

$$M_z(n2) = M_z(0)(\cos\alpha)^{n2} e^{-n2\Delta t/T_1}$$

C.

α = 70°, 90° and 110° give (M_z/M_0) = 0.0137, 0.0000 and 0.0137 in the absence of relaxation.

α = 70°, 90° and 110° give (M_z/M_0) = 0.0374, 0.0165 and 0.0250 in the presence of relaxation.

D.

α = 70°, 90° and 110° give (M_z/M_0) = 0.0068, 0.0000 and 0.0068 in the absence of relaxation.

α = 70°, 90° and 110° give (M_z/M_0) = 0.0008, 0.0000 and 0.0007 in the presence of relaxation.

Note that, besides the 50% signal loss inherent to STEAM, the performance in the absence of relaxation is identical. However, in the presence of T_1 relaxation, STEAM provides better water suppression due to the fact that T_1 relaxation during the TM period is beneficial to the overall suppression.

6.5.

A.

Since CHESS perturbs the longitudinal magnetization, the relevant expression is:

$$M_z(\delta) = \sqrt{M_0^2 - M_{xy}^2(\delta)} = M_0\sqrt{1 - e^{-4(\delta-4.7)^2}}$$

Substituting the chemical shift of creatine gives $M_z(\delta)/M_0$ = 0.955, such that the single CHESS element suppresses the creatine resonance by 4.5% relative the case without any CHESS elements.

B.

The solution derived under A indicates that at the creatine chemical shift position $\cos\theta = M_z/M_0 = 0.955$, or 17.23°. Using Eq. [6.25] the amount of creatine magnetization after six CHESS cycles is given by $(0.955)^6 = 0.7592\ M_0$.

C.

While the perturbation of the creatine resonance by a single CHESS element is small (4.5%), repeated application of CHESS elements in order to improve the water suppression quality also further suppresses the creatine resonance (by 24.1% after 6 CHESS elements). This can pose a problem during spectral quantification when ‘perfect’ water suppression is assumed that leaves the metabolite signals unaffected.

Finite T_1 relaxation actually reduces the unwanted suppression of the creatine resonance as longitudinal magnetization is able to partially recover in between the CHESS elements. This comes of course at the price of a reduced water suppression factor.

6.6.

A.

1. STEAM is based on conventional RF pulses, while LASER is based on adiabatic RF pulses. When the spectra are acquired with a surface coil, the B_1-dependence of the STEAM pulses can lead to signal loss. The B_1-independent, adiabatic RF pulses provide optimal NMR signal, above a minimum threshold B_1 amplitude. These signal losses can not really be eliminated. The average excitation pulses should be carefully optimized, but the spread in nutation angles will always lead to signal loss when using conventional RF pulses.

2. Depending on the exact echo and mixing times, TE and TM, signal from coupled spin-systems can completely cancel during a STEAM sequence (i.e. see Fig. 6.11). This effect does not occur during LASER. This effect can be reduced by using short TE and TM times and when observing non-coupled spin-systems (like water).

3. Magnetic field crusher gradients are essential to the performance of both localization methods. However, the STEAM sequence can generate very large diffusion-weighting factors when strong TE crushers are executed in combination with a long TM period. Water has a large diffusion coefficient and is prone to diffusion-related signal loss. The diffusion time (i.e. the separation between gradients) during a LASER sequence is typically much smaller, leading to much smaller diffusion weighting. This effect can be minimized by using a short TM period and the weakest possible TE crusher gradients that do not compromise the localization quality.

B.

The spectroscopy experiment shows that the localization quality is adequate, such that the observed imaging effect must be artifactual. An effect that is commonly observed is that the linearly varying imaging phase-encoding gradient cancels one of the TE crusher gradients, such that unwanted coherences are not properly removed/dephased. An unwanted echo can often be seen in k-space at the location where this effect occurs. Fourier transformation of k-space then gives the localized volume together with the FFT of the unwanted echo, which often appears as ripples across the image.

This effect can be eliminated by changing the TE crusher gradients, either in direction or amplitude, such that no unwanted echoes are observed within the imaging k-space.

C.

The intrinsic glycine line width based on the T_2 relaxation time is equal to $1/(\pi \times 0.15) = 2.1$ Hz. However, in a relative large volume of 125 μL the observed line width is wider due to residual magnetic field inhomogeneity across the volume (5.0 Hz). Reducing the volume size reduces the amount of residual magnetic field inhomogeneity, thereby leading to a progressively narrower resonance line (3.8 and 2.5 Hz).

In this particular experiment, the line width for the very small volumes increases. This can not be explained in terms of magnetic field homogeneity, but must have a different origin. Reducing a localized volume is normally achieved by increasing the slice selection gradient strength, while keeping the RF bandwidth constant. Stronger gradients generate proportionally stronger eddy currents, which in turn generate spatial and temporal magnetic field variations. Despite gradient pre-emphasis and B_0 correction (see Chapter 10), eddy current compensation is never perfect and as a result residual eddy currents can lead to spectral line broadening.

The effect can be confirmed by acquiring spectra with lower gradient amplitudes and narrower RF pulse bandwidths. While the spatial volume is still the same, the eddy currents should be reduced and the spectral line width should be narrower. Of course in practice a high RF pulse bandwidth is desirable to minimize chemical shift displacement.

In this case, the effects of eddy currents can be corrected post-acquisition as discussed in Chapter 10.

6.7.

A.

Suppose that the ISIS sequence is executed as:

1. no inversion
2. inversion slice 1
3. inversion slice 2
4. inversion slice 1 and 2

The corresponding receiver phase cycle then becomes (1) add, (2) subtract (3) subtract, (4) add.

There are essentially four spatial areas that need to be considered during ISIS, namely:

1. positions that are not affected by either inversion pulse
2. positions that are only affected by the inversion pulse for slice 1
3. positions that are only affected by the inversion pulse for slice 2
4. positions that are affected by both inversion pulses (i.e. the desired volume).

Signal in category 1 is positive during all four scans and is thus subtracted out perfectly. Signal in category 2 appears positive, negative, positive and negative during the four scans, which will thus also lead to perfect cancellation. Signal in category 3 appears positive, positive, negative and negative during the four scans, again leading to cancellation. Only signal in category four, appearing positive, negative, negative and positive, will lead to constructive addition.

B.

The presence of short T_1 relaxation times does not necessarily compromise the localization quality. Short T_1 relaxation times essentially lead to an incomplete inversion, but since two experiments with equally incomplete inversions are subtracted, the cancellation is still perfect. The presence of short T_1 relaxation times does however decrease the maximum detectable signal from the volume-of-interest.

6.8.

A.

For the sequence 90º(+y) – t – βº(+x) – t – acquisition, the magnetization vector at the start of acquisition can be calculated using Eq. [5.8] as:

$$\mathbf{M}_{+x} = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = M_0 \begin{pmatrix} \cos^2 \omega t - \sin^2 \omega t \cos\beta \\ \sin \omega t \cos \omega t (1 + \cos\beta) \\ \sin \omega t \sin\beta \end{pmatrix}$$

Note that the imperfect 180º pulse results in the generation of unwanted M_y and M_z components. For the sequence 90º(+y) – t – βº(+y) – t – acquisition, the magnetization vector at the start of acquisition can be calculated as:

$$\mathbf{M}_{+y} = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = M_0 \begin{pmatrix} \cos^2 \omega t \cos\beta - \sin^2 \omega t \\ \sin \omega t \cos \omega t (1 + \cos\beta) \\ -\cos \omega t \sin\beta \end{pmatrix}$$

Subtracting the two scans, i.e. $\mathbf{M}_{+x} - \mathbf{M}_{+y}$ cancels the M_y component, leaving only the component along the x axis. The M_z component is not important as it can not be detected. Note that the phase cycle only removes unwanted coherences; the decrease in M_x due to incomplete refocusing can not be undone.

B.

For the sequence αº(+y) – t – βº(+x) – t – acquisition, the magnetization vector at the start of acquisition can be calculated as:

$$\mathbf{M}_{+x} = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = M_0 \begin{pmatrix} (\cos^2 \omega t - \sin^2 \omega t \cos\beta)\sin\alpha + \cos\alpha \sin\beta \sin\omega t \\ \sin \omega t \cos \omega t \sin\alpha (1 + \cos\beta) - \cos\alpha \sin\beta \cos\omega t \\ \cos\alpha \cos\beta + \sin\alpha \sin\beta \sin\omega t \end{pmatrix}$$

For the sequence α°(+y) – t – β°(+y) – t – acquisition, the magnetization vector at the start of acquisition can be calculated as:

$$\mathbf{M}_{+y} = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = M_0 \begin{pmatrix} (\cos^2 \omega t \cos\beta - \sin^2 \omega t)\sin\alpha + \cos\alpha \sin\beta \cos\omega t \\ \sin\omega t \cos\omega t \sin\alpha(1+\cos\beta) + \cos\alpha \sin\beta \sin\omega t \\ \cos\alpha\cos\beta - \sin\alpha\sin\beta\cos\omega t \end{pmatrix}$$

Subtracting the two scans, i.e. $\mathbf{M}_{+x} - \mathbf{M}_{+y}$ does in this case NOT cancel the unwanted M_y component. The component of M_y that was excited by the imperfect 180° pulse survives the subtraction process. Extending the phase cycle to +x, +y, –x, –y for the β pulse in concert with a +x, –x, +x, –x receiver phase cycle will remove all M_y components, i.e.:

$$\mathbf{M}_{-x} = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = M_0 \begin{pmatrix} (\cos^2 \omega t - \sin^2 \omega t \cos\beta)\sin\alpha - \cos\alpha \sin\beta \sin\omega t \\ \sin\omega t \cos\omega t \sin\alpha(1+\cos\beta) + \cos\alpha \sin\beta \cos\omega t \\ \cos\alpha\cos\beta - \sin\alpha\sin\beta\sin\omega t \end{pmatrix}$$

and

$$\mathbf{M}_{-y} = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = M_0 \begin{pmatrix} (\cos^2 \omega t \cos\beta - \sin^2 \omega t)\sin\alpha - \cos\alpha \sin\beta \cos\omega t \\ \sin\omega t \cos\omega t \sin\alpha(1+\cos\beta) - \cos\alpha \sin\beta \sin\omega t \\ \cos\alpha\cos\beta + \sin\alpha\sin\beta\cos\omega t \end{pmatrix}$$

Adding and subtracting these scans, i.e. $\mathbf{M}_{+x} - \mathbf{M}_{+y} + \mathbf{M}_{-x} - \mathbf{M}_{-y}$, will give the final signal:

$$\mathbf{M} = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = 2\sin\alpha M_0 \begin{pmatrix} 1-\cos\beta \\ 0 \\ 0 \end{pmatrix}$$

All unwanted coherences have been removed, leaving only a reduced M_x signal.

C.

Since a PRESS sequence contains two independent 180º pulses, the full phase cycle consists of 4 × 4 = 16 steps, namely +x, +y, –x, –y, +x, +y, –x, –y, +x, +y, –x, –y, +x, +y, –x, –y for the first 180º pulse, +x, +x, +x, +x, +y, +y, +y, +y, –x, –x, –x, –x, –y, –y, –y, –y for the second 180º pulse and +x, –x, +x, –x, –x, +x, –x, +x, +x, –x, +x, –x, –x, +x, –x, +x for the receiver.

D. Using the product operator formalism in the spherical tensor basis, the signal at the end of the sequence α°(+y) – t – gradient – β°(+x) – gradient – t – acquisition is given by:

$$\mathbf{M} = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = M_0 \begin{pmatrix} \frac{\sin\alpha}{2}(1-\cos\beta) \\ 0 \\ \cos\alpha\cos\beta \end{pmatrix}$$

This is essentially the same expression as for the full four-step phase cycle, with only two differences, namely (1) the removal of unwanted M_x and M_y components is achieved in a single scan and (2) magnetization that was never excited during either pulse can not be dephased by magnetic field gradients. However, the M_z component is not relevant for the simple spin-echo method under investigation.

6.9.A.

For any stimulated echo sequence a total of eight FIDs, spin- and stimulated echoes can be generated. However with these particular timings, the FIDs of pulses 1 and 2, as well as the spin-echo generated by pulses 1 and 2 (SE12) appear before the final RF pulses. Therefore, only five signals are expected following the last pulse, namely FID3, SE13, SE23, SE123 and STE123.

B.

With the use of three magnetic field gradients of equal area, all signals are destroyed except the STE123 and SE23 signals. As such, the use of equal-area gradients is not optimal in terms of complete removal of unwanted coherences (i.e. SE23).

C.

While the signal from SE23 will be completely dephased in this case, the signal originating from SE123 will now experience no net crusher gradients and can thus potentially be detected.

D.

In addition to the desired signal from the intersection of the three orthogonal slices (STE123), the signal under B will be contaminated with signal from the column that is selected by the second and third RF pulses (SE23). The signal under C will be contaminated with spin-echo signal (SE123) from the same spatial location as the desired STE123 signal.

6.10.

A. The WATERGATE intra-pulse delay τ equals $1/(2\Delta\nu)$, where $\Delta\nu$ represents the off-resonance frequency position for maximum refocusing, and can be calculated as 2.0 ms.

B. A 29.2 μs hard pulse executed with B_1 = 2.0 kHz generates a nutation angle of 21º. Nutation angles of 63º and 133º require pulse length of 87.5 μs and 184.7 μs, respectively.

C. The corrected intra-pulse delay $\tau_{corrected}$ can be calculated as:

$$\tau_{corrected} = \tau - \frac{2T_{pre-\tau}}{\pi} - \frac{2T_{post-\tau}}{\pi}$$

and are given by

1.926 ms, 1.827 ms, 1.765 ms, 1.827 ms and 1.926 ms, respectively.

D.

For a coupled two-spin system, the integrated signal for spin A, $S_{A,total}$, will be the sum of A coherences refocused by the jump-return pulse, S_A, and X coherences that are transferred to the A spins via polarization transfer, $S_{X\rightarrow A}$:

$S_{A,total}(TE, \tau) = S_A(TE, \tau) + S_{X\rightarrow A}(TE, \tau)$

where TE and τ are the echo-time and the JR intra-pulse delay, respectively. S_A and $S_{X\rightarrow A}$ are given by

$$S_A(TE,\tau) = \frac{1}{2}(1 - \cos \pi J\tau \ \ \cos \omega_A \tau) \ \ \cos^2\left(\frac{\pi JTE}{2}\right)$$
$$-\frac{1}{2}(\cos \omega_A \tau - \cos \pi J\tau \ ) \ \ \cos \omega_X \tau \ \ \sin^2\left(\frac{\pi JTE}{2}\right)$$

and

$$S_{X\rightarrow A}(TE,\tau) = \frac{1}{2}\sin \omega_A \tau \ \ \sin \omega_X \tau \ \ \cos\left(\frac{\Delta\omega_{AX}TE}{2}\right)\sin^2\left(\frac{\pi JTE}{2}\right)$$
$$-\frac{1}{2}(\cos \omega_A \tau + \cos \omega_X \tau \ ) \ \ \sin \pi J\tau \ \ \cos\left(\frac{\Delta\omega_{AX}TE}{2}\right)\sin\left(\frac{\pi JTE}{2}\right)\cos\left(\frac{\pi JTE}{2}\right)$$

Graphical evaluation of $S_{A,total}(TE, \tau)$ demonstrates then when ω_{null} and ω_{max} are close or equal to ω_A and ω_X, the sequence acts as a selective spin-echo. However, when spins A and X are both only partially refocused, the observed signal will be dominated by polarization transfer effects.

6.11.

A. Using the product operator formalism as outlined in the appendix, the validity of Eq. [6.9] for a weakly-coupled AX spin-system can be verified.

B. For n = 2 and TE = 1/J, Eq. (6.9) reduces to:

$S(TM) = -(1/4)\cos^2(\pi J TM)\cos(\Delta\omega TM)$

Since $\pi J << \Delta\omega$, the first signal minimum occurs when $\Delta\omega TM = \pi/2$ or $TM = 0.224$ ms. The first signal maximum can be obtained by setting $dS/dTM = 0$ and solving for TM. Alternatively, it can be obtained numerically ($\Delta\omega = 2\pi(4.10 - 1.31).400$, $J = 6.9$ Hz) and is equal to $TM = 0.45$ ms (since $TM = 0$ ms is typically not feasible experimentally).