

8.1.

A.

The optimal delay τ for any CH_n spin-system ($n = 1, 2$ or 3) is equal to $1/(2^1 J_{\text{CH}}) = 3.704$ ms. Since only anti-phase coherences can undergo polarization transfer, the relevant density matrix for a CH spin-system at the time of acquisition is:

$$\sigma = \left(\frac{\gamma_{\text{H}}}{\gamma_{\text{C}}} \right) \sin(\pi^1 J_{\text{CH}} \tau) = 0.9397 \left(\frac{\gamma_{\text{H}}}{\gamma_{\text{C}}} \right)$$

such that the polarization transfer sequence generates 3.747 times more signal than direct detection. Note that the *net* signal following polarization transfer is zero, thereby preventing the use of heteronuclear decoupling for spectral simplification and sensitivity enhancement.

B. The optimal delay τ for the CH spin-system is 3.030 ms. This would lead to a signal intensity of $0.9595(\gamma_{\text{H}}/\gamma_{\text{C}})$ for a CH_2 spin-system.

C.

The relevant expressions for CH and CH_2 groups at the end of a refocused INEPT sequence are given by

$$\sigma_{\text{CH}} = \left(\frac{\gamma_{\text{H}}}{\gamma_{\text{C}}} \right) \sin(\pi^1 J_{\text{CH}} \tau_1) \sin(\pi^1 J_{\text{CH}} \tau_2)$$

$$\sigma_{\text{CH}_2} = \left(\frac{\gamma_{\text{H}}}{\gamma_{\text{C}}} \right) \sin(\pi^1 J_{\text{CH}} \tau_1) \sin(\pi^1 J_{\text{CH}} \tau_2) \cos(\pi^1 J_{\text{CH}} \tau_2)$$

respectively. The optimal delays for CH and CH_2 groups are $\tau_1 = 1/(2^1 J_{\text{CH}})$, $\tau_2 = 1/(2^1 J_{\text{CH}})$ and $\tau_1 = 1/(2^1 J_{\text{CH}})$, $\tau_2 = 1/(4^1 J_{\text{CH}})$, respectively.

A sequence optimized for a CH₂ spin-system with $^1J_{CH} = 135$ Hz is thus executed with $\tau_1 = 3.704$ ms and $\tau_2 = 1.852$ ms. The signal recovery for a CH spin-system with $^1J_{CH} = 165$ Hz is then given by:

$$\sigma_{CH} = 0.7698 \left(\frac{\gamma_H}{\gamma_C} \right)$$

A sequence optimized for a CH spin-system with $^1J_{CH} = 165$ Hz is thus executed with $\tau_1 = \tau_2 = 3.030$ ms. The signal recovery for a CH₂ spin-system with $^1J_{CH} = 135$ Hz is then given by:

$$\sigma_{CH} = 0.2594 \left(\frac{\gamma_H}{\gamma_C} \right)$$

8.2.

From Chapter 2 it follows that the chemical shift positions of GABA-H3 and MM4 are 1.89 and 1.72 ppm, respectively. The frequency difference between these two resonances equals 11, 29 and 51 Hz at 1.5 T, 4.0 T and 7.0 T, respectively. Assuming that GABA-H3 is on-resonance during the editing pulse the nutation angle at the MM4 position equals 159.7°, 77.0° and 13.4° at 1.5 T, 4.0 T and 7.0 T, respectively.

All that remains is to establish a relationship between the editing efficiency and the nutation angle α of the editing pulse for a sequence $90^\circ - t - \alpha^\circ - t - 180^\circ - t - \alpha^\circ - t - \text{acq}$ and a two-spin system AX. Using the product operator formalism it can readily be shown that the density matrix at the start of acquisition is given by

$$\sigma = I_y \cos\alpha \cdot \exp(-TE/T_2)$$

such that the total signal obtained after the two editing steps is given by:

$$I_y(1 - \cos\alpha)\exp(-TE/T_2)$$

Therefore, while the relative GABA signal is 1.271 at all fields, the MM signal varies across the fields as 0.995, 0.398 and 0.014 at 1.5 T, 4.0 T and 7.0 T, respectively. It thus follows that MM contamination is significant at 1.5 and 4.0 T, but that it becomes negligible at magnetic fields of 7.0 T and higher. Of course it should be realized that the exact numbers are affected by T_2 relaxation, pulse shape and spin-system under investigation. Nevertheless, the above calculation can serve as a general recipe to obtain theoretical signal recovery estimates.

8.3.

A. For J-difference editing, as well as MQC-based editing (utilizing both ZQCs And DQCs), the signal at the start of acquisition (after 2 averages) is given by:

$$M_{xy}(TE) = M_{xy}(0)[1 - \cos \pi JTE] e^{-TE/T_2}$$

B. The optimal echo-time gives the maximum amount of lactate signal and can be obtained by setting the derivative of $M_{xy}(TE)$ with respect to TE to zero and is given by:

$$TE_{\text{optimal}} = \frac{1}{\pi J} \arccos\left(\frac{1 - \pi^2 J^2 T_2^2}{1 + \pi^2 J^2 T_2^2}\right)$$

For a T_2 relaxation time constant of 100 ms and $J = 6.9$ Hz, the optimal echo-time then equal 105 ms.

C. Using an echo-time of 105 ms gives 21.7% more signal than the standard echo-time of $1/J = 144$ ms.

D. For lactate editing, the total echo-time from the middle of the excitation pulse to the beginning of acquisition is 144 ms. In the presence of a $TE_1 = 20$ ms delay surrounding

the first 180° pulse, the selective editing pulses only refocus scalar coupling over the last TE2 = 124 ms. The expression for the signal intensity at the time of acquisition is then equal to:

$$M_{xy}(TE) = M_{xy}(0)[\cos \pi JTE1 - \cos \pi JTE] e^{-TE/T_2}$$

This gives circa 4.6% less signal as compared to the situation where TE1 = 0 ms and TE2 = 144 ms.

E. GABA-H4 can be seen as an AX₂ spin-system for which the edited signal intensity at the start of acquisition is given by:

$$M_{xy}(TE) = M_{xy}(0)[\cos^2(\pi JTE1) - \cos^2(\pi JTE)] e^{-TE/T_2}$$

which reduces to

$$M_{xy}(TE) = M_{xy}(0)[1 - \cos^2(\pi JTE)] e^{-TE/T_2}$$

for TE1 = 0. Therefore, the edited GABA signal intensity with the additional delay is 19.7% lower as compared to the situation where TE1 = 0 ms and TE2 = 68 ms. The signal loss effect is much larger for GABA because the 20 ms is a larger fraction of the total echo time of 68 ms. It is therefore imperative that any delays that are not directly associated with the spectral editing step be minimized.

8.4. [3-¹³C]-lactate can be considered a I₃S spin-system with J_{IS} = 127.5 Hz.

A. When t₁ = 1/(2J), the in-phase signal at the time of acquisition is given by:

$$M_{xy}(t_2) = M_{xy}(0) \sin(\pi Jt_2) \cos^2(\pi Jt_2)$$

The highest amount of signal is then obtained for $t_2 \sim 1/(5J)$ and equals $\sim 0.385M_{xy}(0)$.

B.

The signals for direct detection and detection through polarization transfer are given by:

$$M_{xy, \text{direct}} = M_0 (1 - e^{-TR/T_{1c}}) \text{ and}$$

$$M_{xy, \text{PT}} = M_0 \left(\frac{\gamma_H}{\gamma_C} \right) (1 - e^{-TR/T_{1H}}) e^{-t_1/T_{2H}} e^{-t_2/T_{2c}} \sin(\pi J t_1) \sin(\pi J t_2) \cos^2(\pi J t_2)$$

With the optimal t_1 and t_2 delays this results in:

$$M_{xy, \text{direct}} = 0.811 M_0 \text{ and } M_{xy, \text{PT}} = 1.094 M_0$$

Therefore, even though polarization transfer gives a factor of (γ_H/γ_C) for an AX spin-system, the inefficient generation of in-phase coherences for an AX₃ spin-system greatly reduces this advantage.

8.5. Repeated application of Eqs. (A4.10) and (A4.11) in the appendix is all that is needed to derive Eqs. (8.9) – (8.12).

For example, in an IS₂ spin-system first consider the scalar coupling evolution of spin I with respect to the first S spin, S₁:

$$I_x \longrightarrow I_x \cos(\pi J_{IS} t) + 2I_y S_{1z} \sin(\pi J_{IS} t)$$

Next expand this expression by considering the scalar coupling evolution of spin I with respect to the second S spin, S₂:

$$I_x \longrightarrow [I_x \cos(\pi J_{IS} t) + 2I_y S_{2z} \sin(\pi J_{IS} t)] \cos(\pi J_{IS} t) + 2[I_y \cos(\pi J_{IS} t) - 2I_x S_{2z} \sin(\pi J_{IS} t)] S_{1z} \sin(\pi J_{IS} t)$$

Expanding this expression then gives Eq. (8.9):

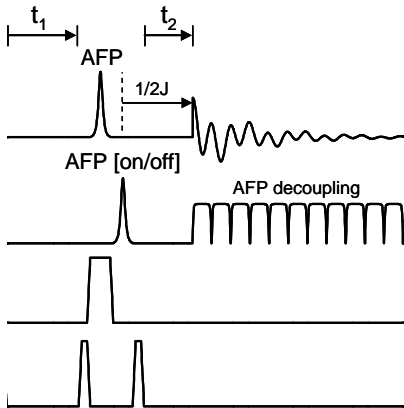
$$I_x \longrightarrow I_x \cos^2(\pi J_{IS} t) + 2I_y (S_{1z} + S_{2z}) \sin(\pi J_{IS} t) \cos(\pi J_{IS} t) - 4I_x S_{1z} S_{2z} \sin^2(\pi J_{IS} t)$$

Eqs. (8.10) – (8.12) can be derived using similar arguments.

8.6.

A.

The relevant delays, t_1 and t_2 , are indicated:



Only two criteria are important for the calculation of t_1 and t_2 , namely (1) that the duration from the middle of the ^{13}C AFP pulse to the start of acquisition equals $1/(2J)$ and that (2) the ^1H AFP pulse refocuses all chemical shifts and B_0 offsets, which means that the delays on either side of the ^1H 180° pulse must be identical.

t_2 can be calculated as:

$t_2 = 1/(2J) - \text{gradient duration} - T_{\text{AFP-}^{13}\text{C}}/2$, whereby $T_{\text{AFP-}^{13}\text{C}}$ equals the length of the ^{13}C AFP pulse and the gradient duration equals two gradient ramp durations and a gradient plateau duration.

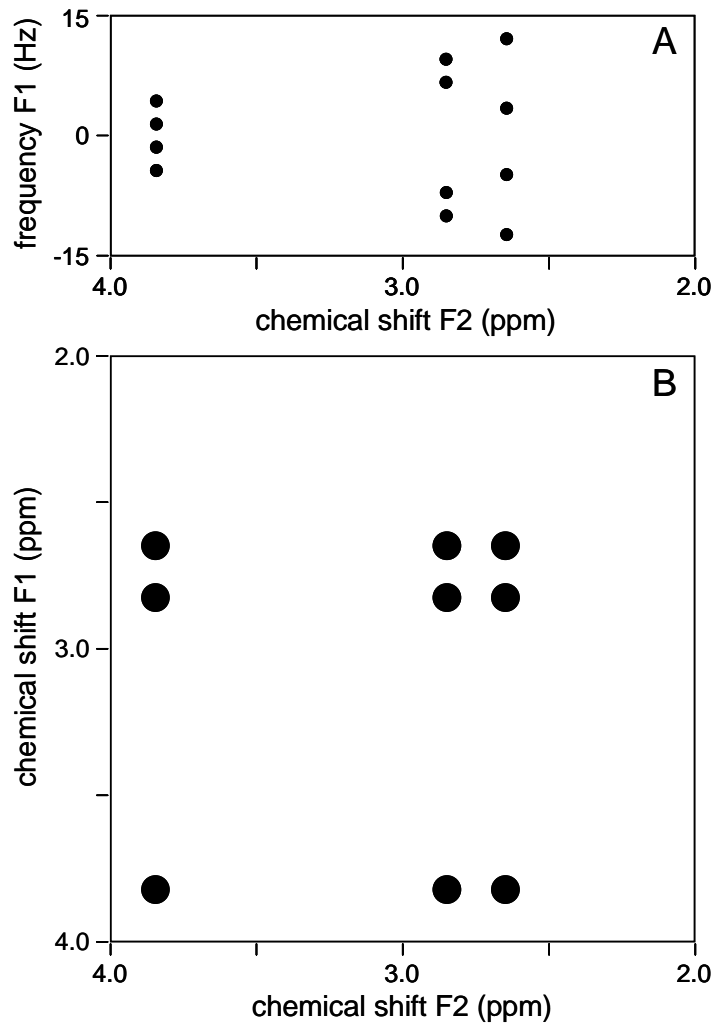
t_1 can be calculated from t_2 as to keep the delays surrounding the ^1H AFP pulse symmetrical:

$$t_1 = t_2 + T_{\text{AFP-}^{13}\text{C}}$$

B.

^{13}C MRS is characterized by a very wide chemical shift range. When a ^{13}C frequency-selective RF pulse is executed during a magnetic field gradient, the wide chemical shift range leads to large spatial displacements. This can lead to the situation that the ^{13}C 180° pulse does not invert the ^{13}C magnetization in all locations selected by the ^1H 180° pulse, thus leading to signal loss in the final, edited spectrum.

8.7.



8.8.A. Lactate is a scalar-coupled spin-system. Increasing the echo-time in the absence of spectral editing pulses will lead to regular scalar coupling evolution in which the detectable signal is proportional to $M_{xy}(0)\cos(\pi JTE)\cdot\exp(-TE/T_2)$ instead of $M_{xy}(0)\exp(-TE/T_2)$. The cosine function can lead to an unexpectedly short T_2 relaxation time constant when the measured data is fitted with $\exp(-TE/T_2)$.

B.

1. Fitting of the measured data with $M_{xy}(0)\cos(\pi JTE)\cdot\exp(-TE/T_2)$. Since J and TE are quantitatively known, the fit only has 2 unknowns, $M_{xy}(0)$ and T_2 .
2. Detect lactate with a sequence that selectively refocuses the lactate-H3 protons. The selective refocusing inhibits scalar coupling evolution.
3. Detect lactate with a regular spin-echo at a multiple of $2/J$. Since lactate is positive at these echo-times, the signal-vs-TE curves can simply be fit with a single exponential function.
4. When the exact, 3D magnetic field inhomogeneity across a localized volume is known (for example from a 3D B_0 map), the B_0 -related line broadening can be subtracted from the lactate resonance, such that the remaining line width is inversely proportional to T_2 .

8.9.

A.

The creatine methyl signal resonates at 3.03 ppm, whereas the GABA-H3 multiplet appears at 1.89 ppm. Therefore, GABA-H3 has a resonance frequency that is 342.1 Hz lower than that of creatine, or 844.2 Hz lower than that of water. Therefore, the carrier frequency of the spectral editing pulse must be 300.141158 MHz

B. The macromolecular M4 resonance appears at 1.72 ppm, or -51 Hz away from GABA-H3. The carrier frequency for the mirror position would therefore equal 1.55 ppm or 300.141056 MHz.

8.10.A. The density matrices at the end of a BISEP pulse for uncoupled and heteronuclear scalar coupling spin systems are given by:

$\sigma = I_z$ and $\sigma = I_z \cos(2\pi Jt) - 2I_x S_z \sin(2\pi Jt)$, respectively. For $t = 1/(4J)$ the heteronuclear coupled spins are thus fully excited as anti-phase coherence $-2I_x S_z$, whereas the uncoupled spins remain along the z axis.

B.

By inserting an additional segment, $1/4J - 90^\circ_{+x} (^1\text{H}) - 2t - 180^\circ_{-x} (^1\text{H}/^{13}\text{C}) - 2t - 90^\circ_{+x} (^1\text{H}) - 1/4J$, the sequence is effectively transformed into a BISEP spin-echo in which uncoupled spins are not excited or refocused and in which heteronuclear scalar coupled spins are excited as anti-phase coherences, but evolve into in-phase coherences at the top of the spin-echo.

8.11. The magnetic field gradients surrounding the two editing pulses are fairly close together during the short-echo-time MRS scan. This likely leads to a small diffusion-weighting factor b and hence only a small amount of diffusion-related signal loss. However, when the editing pulses are turned on, the gradient separation Δ becomes much larger (at least 30 ms), such that the diffusion-weighting significantly increases. This can potentially lead to increased diffusion-related signal loss.