10.1.

A.

A shim change ΔZ = 250 Hz/cm generates a change in Z^0 of (250/1000)×20 = 5 Hz, a shim change ΔZ^2 = –450 Hz/cm^2 generates a change in Z^0 of (–450/500)×200 = –180 Hz and a shim change ΔZ^3 = 60 Hz/cm^3 generates a change in Z^0 of (60/80)×150 = 112.5 Hz.

The total Z^0 field shift is therefore –62.5 Hz, which can be corrected with a +1.250% Z^0 shim change.

B.

A shim change ΔZ^3 = 60 Hz/cm^3 generates a change in Z^0 of (60/80)×150 = 112.5 Hz. In addition, the shim change ΔZ^3 = 60 Hz/cm^3 generates a change in Z of (60/80)×–40 = –30 Hz/cm. The total ΔZ shim change thus becomes 250 – (– 30) = 280 Hz/cm, which generates a (280/1000)×20 = 5.6 Hz.

The total Z^0 field shift is therefore –61.9 Hz, which can be corrected with a +1.238% Z^0 shim change.

C.

A Z^3 magnetic field inhomogeneity out of the isocenter is equal to:

$$\Delta B_0(x, y, z) = 2(z_0 + z)^3 - 3(z_0 + z)[(x_0 + x)^2 + (y_0 + y)^2]$$

which can be simplified to a combination of zero through third order functions given by:

$$2z^3 - 3z(x^2 + y^2)$$

$$3z[2z_0^2 - (x_0^2 + y_0^2)] - 6xx_0z_0 - 6yy_0z_0$$

$$3[2z^2 - (x^2 + y^2)]z_0 - 6xzx_0 - 6yzy_0$$

$$2z_0^3 - 3z_0(x_0^2 + y_0^2)$$

Therefore, a pure Z^3 spherical harmonic function measured out of the magnet isocenter requires 8 different shim changes to achieve full cancellation. Substitution of (x0, y0, z0) = (+1, –2, +3) gives the following relative shim changes:

$Z^3 = +1$

$Z = +39$, $X = -18$, $Y = +36$

$Z^2 = +9$, $XZ = -6$, $YZ = +12$

$Z^0 = +9$

10.2.

A. By using Table 10.2 and setting Y = constant, the non-degenerate shims for the coronal plane are given by:

$X, Z, Z^2, ZX, X^2 - Y^2, Z^3, Z^2X, Z(X^2 - Y^2), X^3$

B. By using Table 10.2 and setting X = constant, the non-degenerate shims for the sagittal plane are given by:

$Y, Z, Z^2, ZY, X^2 - Y^2, Z^3, Z^2Y, Z(X^2 - Y^2), Y^3$

10.3.

A. Using Eq. [10.51] and solving $B_{1ax}(y, r) = B_{1ax}(0, r)/3$, the distance r is equal to 7.27 mm.

B.

It is not possible to generate a B_1 field of 0.58 Gauss 10 mm away from the coil for any radius without increasing the current. This is because the efficiency of the coil to generate a B_1 field decreases with increasing coil radius, so that the current must increase in order to maintain the same B_1 field.

10.4.

A.

$$B_0(t) = \sum_{k=1}^{n} B_{0,k}[e^{-t/TC_{B0,k}} - e^{-(t+T)/TC_{B0,k}}]$$

whereby the second term represents the effect up the rising gradient edge. Note that the effect/sign is opposite to that achieved for the falling gradient edge. Further note that for an infinitely long gradient pulse ($T \rightarrow \infty$), the equation reduces to Eq. [10.29]. A similar expression can be formulated for the gradient term.

B.

$$B_0(t) = \sum_{k=1}^{n} B_{0,k}[e^{-t/TC_{B0,k}} - 2e^{-(t+T)/TC_{B0,k}} + e^{-(t+2T)/TC_{B0,k}}]$$

C.

For gradient pulses of T = 2 ms, eddy currents with TC ≥ 5.26 ms will be suppressed for ≥ 10%. This method can be very effective in removing eddy current effects with very long time constants (>> 100 ms). However, for many NMR experiments it is not feasible or desirable to have a {+G, –G} gradient combination.

D.

Assuming that the pre-emphasis and B_0 compensation have been properly adjusted, a likely explanation for the observed phenomenon is that the gradient coil assembly has significant gradient cross terms (e.g. eddy currents in the X direction generated by a gradient applied in the Y direction). Regular pre-emphasis is applied on the same gradient channel (i.e. X pre-emphasis on a X-gradient). However, in the presence of significant cross terms, the pre-emphasis must also be applied to orthogonal channels (i.e. X pre-emphasis on a Y-gradient). This can easily be verified with the experimental setup detailed in Figs. 10.19 and 10.20.

10.5.A.

Using Eq. [10.61] gives λ = 13.26 cm.

B.

Using Eq. [10.61] gives a skin depth of 6.1 cm.

10.6.A. Derive expressions for the spherical harmonic fields generated by the five second-order shim coils in the reference frame of the magnet.

The transformation for a 2D xy rotation is given by

$x' = x\cos(15^{o}) - y\sin(15^{o}) = 0.9659x - 0.2588y$

$y' = x\sin(15^{o}) + y\cos(15^{o}) = 0.2588x + 0.9659y$

whereas the transformation for a 1D z translation is given by

$z' = z + 20$

The functional forms of the actual shim fields produced by the first and second order shims then become

$X' = 0.9659x - 0.2588y$

$Y' = 0.2588x + 0.9659y$

$Z' = z + 20$

$Z^{2}{}' = 2(z + 20)^2 - (0.9659x - 0.2588y)^2 - (0.2588x + 0.9659y)^2$ or

$Z^{2}{}' = 2z^2 - (x^2 + y^2) + 80z + 800$

$ZX' = (z + 20)(0.9659x - 0.2588y) = 0.9659zx - 0.2588zy + 19.32x - 5.176y$

$ZY' = (z + 20)(0.2588x + 0.9659y) = 0.2588zx + 0.9659zy + 5.176x + 19.32y$

$XY' = (0.9659z - 0.2588y)(0.2588x + 0.9659y) = 0.25(x^2 - y^2) + 0.866xy$

$X^2\text{-}Y^{2}{}' = (0.9659x - 0.2588y)^2 - (0.2588x + 0.9659y)^2 = 0.866(x^2 - y^2) - 0.9999xy$

Note that the radial symmetry of the Z and Z^2 shims makes them invariant to x-y rotations.

B. Magnetic field mapping reveals a magnetic field distribution given by $+20Z^2 + 33XZ + 10Y$. Calculate all first and second order shim corrections necessary to achieve a homogenous magnetic field.

In order to compensate the magnetic field inhomogeneity, first consider the second order shims. Ignoring, for the moment, the first order contributions to ZX' and ZY' it can be shown that a pure ZX shim can be obtained with the combination 0.9659ZX' + 0.2588ZY'. Therefore, the +33ZX inhomogeneity can be compensated with relative currents in the ZX' and ZY' shim coils of –31.88 and –8.54, respectively. The $+20Z^2$ inhomogeneity can simply be compensated with a relative current of –20 in the Z^2' shim coil.

However, the currents in the second-order shim coils produce first order shim imperfections, given by –615.8X + 165.0Y, –44.2X – 165.0Y and – 1600Z for the ZX', ZY' and Z^2' shim coils.

The total first order magnetic field inhomogeneity that should be compensated is then equal to –660X + 10Y – 1600Z.

Ignoring zero-order effects, pure X, Y and Z shim field can be generated with the combinations 0.9659X' + 0.2588Y', –0.2588X' + 0.9659Y' and Z', respectively. Therefore, the first order inhomogeneity can be cancelled by putting relative currents of 660*0.9659 – 10*–0.2588 = 640, 660*0.2588 – 10*0.9659 = 161.2 and +1600 in the X', Y' and Z' shim coils respectively.

10.7A.

The loading characteristics of a bottle of distilled water will be very different from the human head, such that it is likely that the coil will need significant adjustment of both the tuning and matching capacitors.

B. The proton resonance frequency at 3.0 T equals 127.7 MHz. The coil therefore needs to be tuned by +4.3 MHz.

C. A small drop in Q value upon insertion of a non-lossy sample such as a solid plastic phantom is an indication of small electrical fields, which is a desirable feature for any RF coil.

D. A large drop in Q value upon insertion of a lossy sample such as a human head is indicative of an efficient interaction between the coil and the sample and is a desirable feature for sensitive NMR detection and efficient RF transmission.

E. The relative drop of the Q value upon loading the coil with a sample is identical for both coil ($\Delta Q = 60\%$). In that case, the coil with the highest Q value is preferable as the sensitivity is proportional to the Q value of the coil.

10.8.

A. Using Eq. [10.21] gives $t_{ramp} = 171.4$ μs (Note: 1 H = 1 m^2 kg s^{-2} A^{-2} and 1 V = 1 m^2 kg s^{-3} A^{-1}).

B. $G = \eta I = 45$ mT/m.

C. $t_{ramp} = 133.3$ μs and G = 60 mT/m.

D. The higher gradient amplitude allows signal acquisition over a wider imaging bandwidth, whereas the shorter ramp time maximizes the data acquisition duty cycle (i.e. there is less time over which no data is acquired). Both mechanisms shorten the acquisition window of the EPI echo-train, which will lead to reduced image distortions in the phase-encoding ('blip') direction.

10.9.

A.

Using Table 10.1, R = 1.5 cm, (x, y, z) = (0, 0, 5) cm and $\Delta\chi = 9.42$ ppm = 2826 Hz, the magnetic field offset in the frontal cortex at 7.05 T is equal to 50.9 Hz.

B.

When the magnetic field direction is changed to the ear – ear orientation, the (x, y, z) labels in Table 10.1 must be changed with (z, y, x) labels. As a result, the magnetic field offset reduces by a factor of –2 to –25.4 Hz. This confirms that the orientation of the main magnetic field can have a significant effect on the encountered magnetic field homogeneity.

C.

The main consideration involved is the fact that open MR magnet designs are limited to low magnetic field strengths. The issue of magnetic field inhomogeneity is not a dominant factor at the lower magnetic fields available for open MR magnets. All high field MR magnets are of a solenoidal design, making the proposed change in magnetic field orientation impossible.

D.

The magnetic field offset for air was calculated under A and is equal to 50.9 Hz. When the sphere of air is replaced with a sphere of nitrogen, the offset changes to 48.8 Hz which constitutes a reduction of only 4.1%.

E.

$\chi_{expiration} = 0.18 \times 1.8 + 0.82 \times -0.0063 = 0.3188$ ppm = 95.65 Hz

$\chi_{inspiration} = 0.22 \times 1.8 + 0.78 \times -0.0063 = 0.3911$ ppm = 117.33 Hz

Using Table 10.1, R = 9.5 cm or 11.5 cm and (x, y, z) = (0, 0, 30) cm, the change in magnetic field between inspiration (ΔB_0 = 4.41 Hz) and expiration (ΔB_0 = 2.02 Hz) can be calculated as 2.38 Hz at the position of the human brain.

F.

A frequency variation of 2.38 Hz leads to a phase variation of 21.4º during an echo-time TE of 25 ms.

G.

While a phase variation of 21.4º does not appear very large, it can in fact lead to significant image blurring and fMRI image artifacts. The respiration-induced phase variation can be minimized by using navigator-echoes, respiration-triggering or even dynamic adjustment of the B_0 magnetic field.

10.10. During phase-sensitive quadrature detection one channel produces a signal with a relative amplitude and phase of 100 and 20º, whereas the other channel produces a signal with a relative amplitude and phase of 104 and 110º.

A.

Without a receiver imbalance, the time domain signal can be described as:

$$S(t) = S(0)\cos\omega t + iS(0)\sin\omega t$$

In the presence of a receiver imbalance, the time domain signal can be described as:

$S(t) = S(0)\cos\omega t + 1.04iS(0)\sin\omega t$, which can be expanded according to:

$$S(t) = S(0)\cos\omega t + iS(0)\sin\omega t + 0.04iS(0)\sin\omega t$$

$$= S(0)\cos\omega t + iS(0)\sin\omega t + 0.02S(0)[(\cos\omega t + i\sin\omega t) - (\cos(-\omega t) + i\sin(-\omega t))]$$

or

$$S(t) = 1.02S(0)[\cos\omega t + i\sin\omega t] - 0.02S(0)[\cos(-\omega t) + i\sin(-\omega t)]$$

The receiver imbalance thus leads to an unwanted 'ghost' signal at the negative frequency of the main signal with an amplitude that is –2% of the original signal.

B. In the presence of a phase imbalance, the time domain signal can be described as:

$$S(t) = S(0)\cos\omega t + iS(0)[\sin\omega t\sin\alpha + \cos\omega t\cos\alpha]$$

where α is the angle between the real and imaginary channels. $\alpha = 90^\circ$ for a perfectly balanced receiver. For $\alpha = 95^\circ$, the expression becomes

$$S(t) = S(0)\cos\omega t + iS(0)[0.9962\sin\omega t - 0.0872\cos\omega t]$$

This expression can be expanded according to:

$$S(t) = S(0)\cos\omega t + iS(0)\sin\omega t - 0.038iS(0)\sin\omega t - 0.0872iS(0)\cos\omega t$$

Expanding the final two terms, gives the expression:

$$S(t) = 0.981S(0)[\cos\omega t + i\sin\omega t] + 0.019S(0)[\cos(-\omega t) + i\sin(-\omega t)] - \ldots$$

$$0.0436iS(0)[\cos\omega t + i\sin\omega t] - 0.0436iS(0)[\cos(-\omega t) + i\sin(-\omega t)]$$

The receiver phase imbalance thus leads to an unwanted 'ghost' signal at the negative frequency of the main signal with a mixed phase. Furthermore, the original signal at frequency $+\omega$ also has a small out-of-phase contribution.

C. A constant offset (i.e. a DC offset) on both channels can be described by:

$$S(t) = A + iB$$

which following Fourier transformation would give rise to a signal a $\omega = 0$.